Solver

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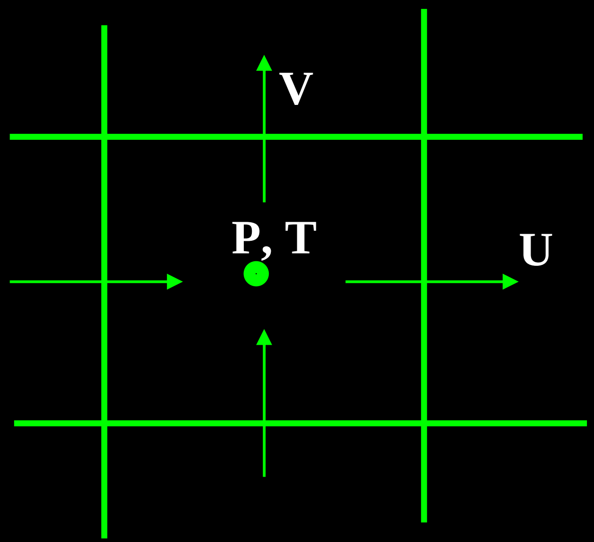
Solver

- Discretized equations must be solved
- Coefficients in equations are coupled
- e.g. Velocities are influenced by pressure gradients
- e.g. Velocities are influenced by temperature gradients (buoyancy)
- Must therefore solve iteratively

Solver

- Pressure and Temperature are evaluated at Cell Centres
- Velocities are evaluated at Cell Faces



Solver

- Staggered Grid
- Gives correct pressure gradient terms in momentum equations
- Velocities are evaluated where needed for convection-diffusion equations

Solver

- At start of solution, velocity field and pressure field are unknown
- FLOTHERM uses SIMPLEST algorithm to evaluate coupled velocity and pressure field
- SIMPLEST is based on the Semi-Implicit Method for Pressure-Linked Equations (developed by Patankar and Spalding)

Solver

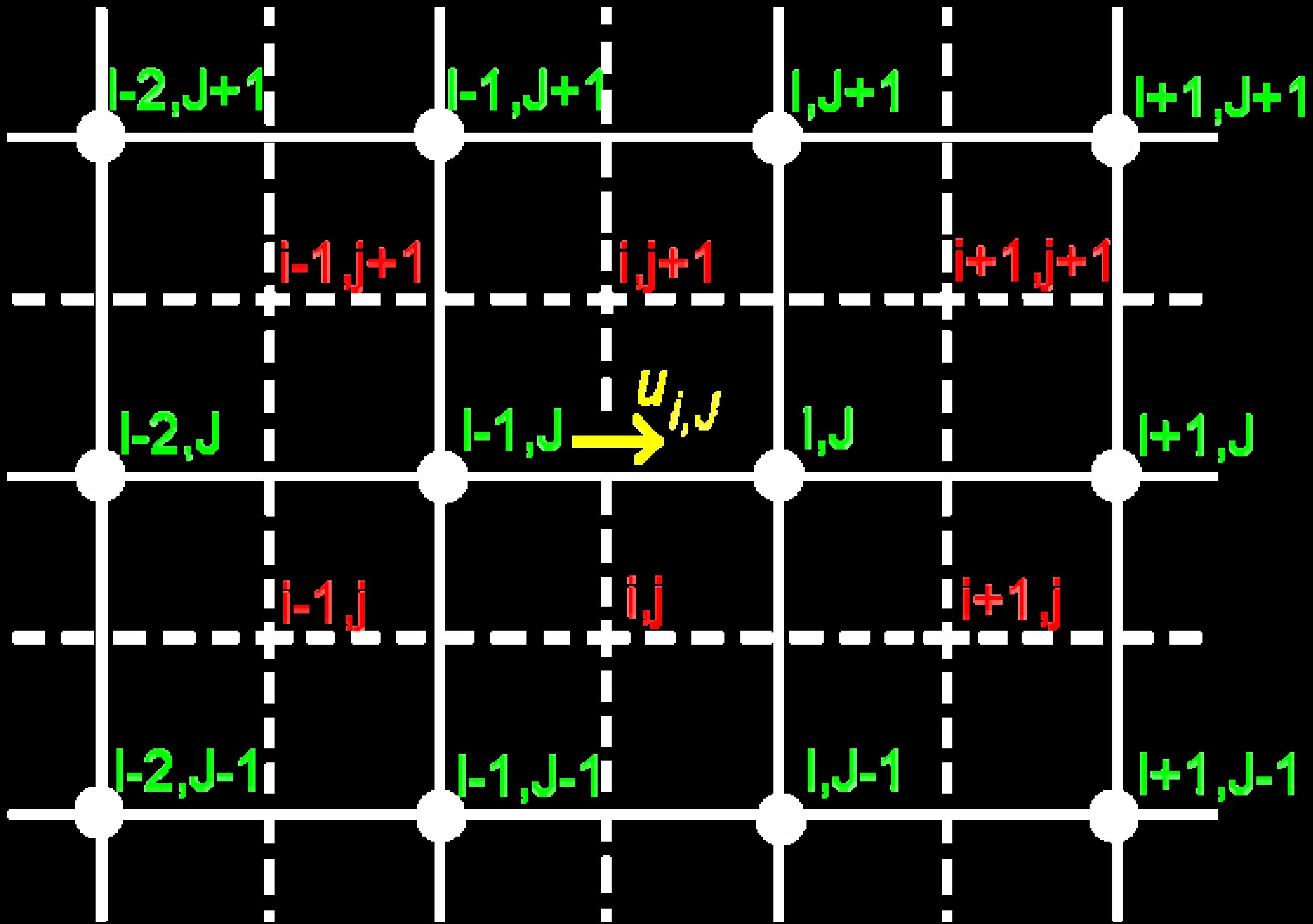
- Consider 2-D Laminar flow:

$$\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu)=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}+S_u$$

$$\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho vv)=\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)-\frac{\partial p}{\partial y}+S_v$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

Solver


I-2,J+1
I-1,J+1
I,J+1
I+1,J+1
i-1,j+1
i,j+1
i+1,j+1
$u_{i,J}$
I-2,J
I-1,J
I,J
I+1,J
i-1,j
i,j
i+1,j
I-2,J-1
I-1,J-1
I,J-1
I+1,J-1

Solver

- Using the staggered grid, we can write

$$a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} - \frac{p_{I,J} - p_{I-1,J}}{\delta x_U} \Delta V_u + \bar{S} \Delta V_u$$

- This is the discretized momentum equation, which links the pressure and velocity fields

Solver

- We can write:

$$a_{i,J}u_{i,J} = \sum a_{nb}u_{nb} + \left(p_{I-1,J} - p_{I,J}\right)A_{i,J} + b_{i,J}$$

- With $b_{i,J}$ as the momentum source term

Solver

- Solution of the equations proceeds as follows:
- A pressure field p* is guessed
- The discretized momentum equation is solved, to give velocity components, u* and v*

Solver

- Solving the momentum equation:

$$a_{i,J}u^{*}_{i,J} = \sum a_{nb}u^{*}_{nb} + \left(p^{*}_{I-1,J} - p^{*}_{I,J}\right)A_{i,J} + b_{i,J}$$

- Now, define correction p' as the difference between the correct pressure field, p and the guessed field, p*

Solver

- Velocity corrections are defined in the same way:

$$p = p^* + p'$$

$$u = u^* + u'$$

$$v = v^* + v'$$

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Solver

- Of course, we do not know the correct pressure and velocity fields
- But, we know that they are related by the momentum equation:

$$a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} + \left(p_{I-1,J} - p_{I,J}\right) A_{i,J} + b_{i,J}$$

Solver

- Subtracting the correction form of the equation from correct form:

$$a_{i,J}\left(u_{i,J} - u^*_{i,J}\right) = \sum\left(a_{nb}u_{nb} - a_{nb}u^*_{nb}\right) + \left(\left[p_{I-1,J} - p^*_{I-1,J}\right] - \left[p_{I,J} - p^*_{I,J}\right]\right)A_{i,J}$$

Solver

- In terms of corrections:

$$a_{i,J}u'_{i,J} = \sum a_{nb}u'_{nb} + \left(p'_{I-1,J} - p'_{I,J}\right)A_{i,J}$$

- We now ignore the term

$$\sum a_{nb}u'_{nb}$$

Solver

- Finally the velocity correction is:

$$a_{i,J}u'_{i,J} = \left(p'_{I-1,J} - p'_{I,J}\right)A_{i,J}$$

- So the velocity is given by:

$$u_{i,J} = u^*_{i,J} + \frac{\left(p'_{I-1,J} - p'_{I,J}\right)A_{i,J}}{a_{i,J}}$$

Solver

- Our second constraint on the velocity field is that it must satisfy continuity
- We use this constraint to determine the pressure correction, p’

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Solver

Solver

- Continuity balance on cell:

$$\left[(\rho u A)_{i+1,J} - (\rho u A)_{i,J}\right] + \left[(\rho u A)_{I,j+1} - (\rho u A)_{I,j}\right] = 0$$

- We substitute back the corrected velocities into this equation

$$(\rho A u)_{i,J} = \rho_{i,J} A_{i,J}\left(u^{*}_{i,J} + \frac{\left(p'_{I-1,J} - p'_{I,J}\right)A_{i,J}}{a_{i,J}}\right)$$

Solver

- We can get an expression for p’ at the cell centre:

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,J+1}p'_{I,J+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J}$$

- With: $a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1}$
- The source term is the continuity imbalance arising from the incorrect velocity field

Solver

- The coefficients are given by:

$$a_{I+1,J} = \left(\frac{\rho A^2}{a}\right)_{i+1,J} \qquad a_{I-1,J} = \left(\frac{\rho A^2}{a}\right)_{i,J}$$

$$a_{I,J+1} = \left(\frac{\rho A^2}{a}\right)_{I,j+1} \qquad a_{I,J-1} = \left(\frac{\rho A^2}{a}\right)_{I,j}$$

Solver

- For a converged solution

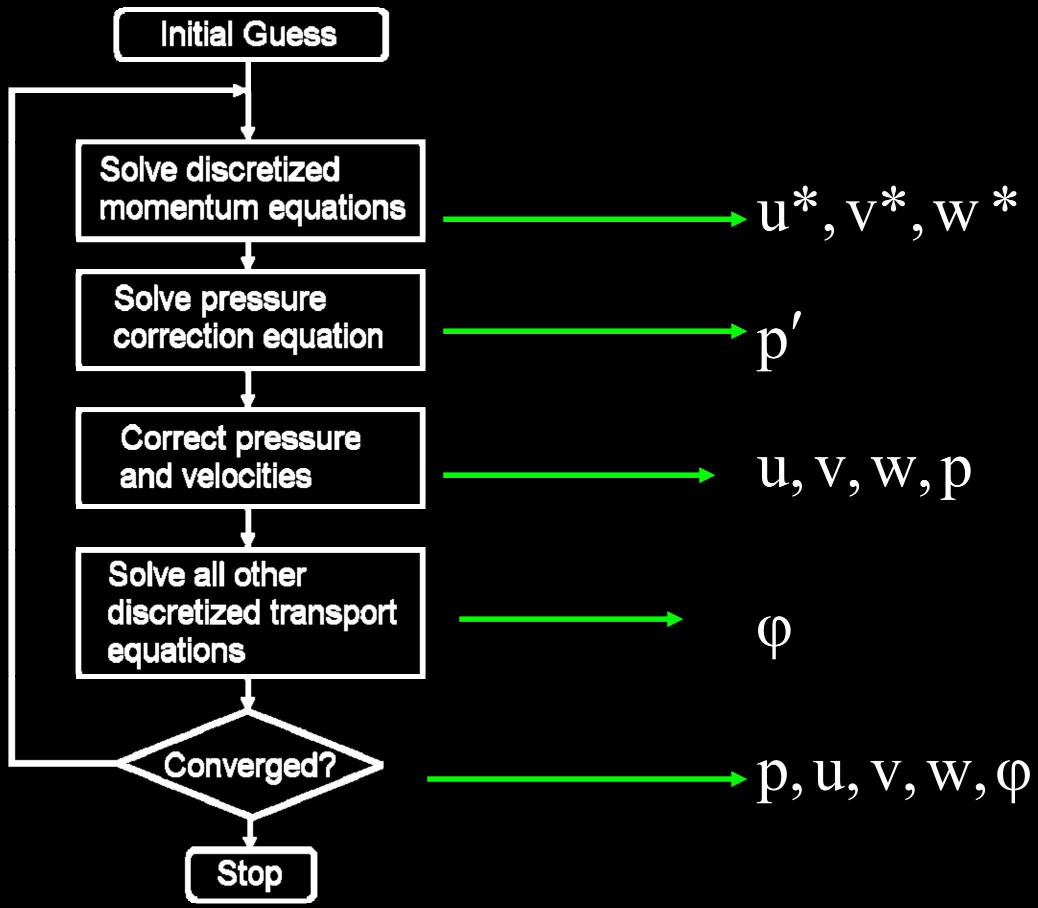
$$p = p^*$$

$$u = u^*$$

$$v = v^*$$

- With all the corrections equal to zero

Solver


Initial Guess
Solve discretized momentum equations
u*, v*, w *
Solve pressure correction equation
p′
Correct pressure and velocities
u, v, w, p
Solve all other discretized transport equations
φ
Converged?
p, u, v, w, φ
Stop

Solver

- Solver continues until the imbalances in the equations are less than that specified by the termination criteria
- There are two methods for controlling convergence:
- Linear relaxation
- False time step

Solver

- Linear relaxation controls the amount of correction that is used at each iteration

$$p^{new} = p^{*} + \alpha_{p} p'$$

$$u^{new} = \alpha_{u} u + (1 - \alpha_{u}) u^{(n-1)}$$

- By default, no linear relaxation is used in FLOTHERM

Solver

- False time step
- In terms of corrections:

$$a_{i,J} u'_{i,J} = \left(p'_{I-1,J} - p'_{I,J}\right) A_{i,J}$$

- Once the solution has converged, the corrections are equal to zero
- Therefore, the central coefficient may be adjusted

Solver

- The central coefficient is adjusted using the false time step
- Mass of cell / false time step is added to central coefficient

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False Time Step

- False time step should be based on physical time scale of problem
- Residence time in domain
- Residence time in average cell

Solver

- Residence time in domain = Mean domain size / average velocity
- Residence time in cell = Mean cell size / average velocity
- Normally, will set false time step within these limits

Solver

- Because pressure correction comes from the continuity equation, no false time step for pressure
- Can use linear relaxation instead
- Or use false time steps for velocities

Solver

- Conduction only problems, no false time step is needed
- Should solve in 1 outer iteration
- More efficient to limit inner iterations and perform several outer iterations

Linear Solver

- The discretized equations for pressure, velocity and temperature must be solved
- Need an linear solver to do this
- FLOTHERM offers three options

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Linear Solver

- Segregrated Solver
- Segregrated Conjugate Residual
- Multigrid

Linear Solver

- Segregated:
- Uses Gauss-Seidel solver
- Looks locally at flow field
- Can be slow to capture trends

Linear Solver

- Segregrated Conjugate Residual
- Based on Conjugate Gradient method
- Considers whole-field interactions
- Fast at resolving trends through whole domain

Linear Solver

- Multigrid
- Same as segregrated conjugate residual for velocities and pressure
- For temperature, solves on progressively coarser grid
- Potentially very fast to converge temperature field; removes need for block correction

Solver

- Block correction
- Splits domain into blocks
- Generates new discretized equations for each block
- Solves equations
- Should give fast convergence for systems with large conductivity variations