Boundary Layers and Heat Transfer

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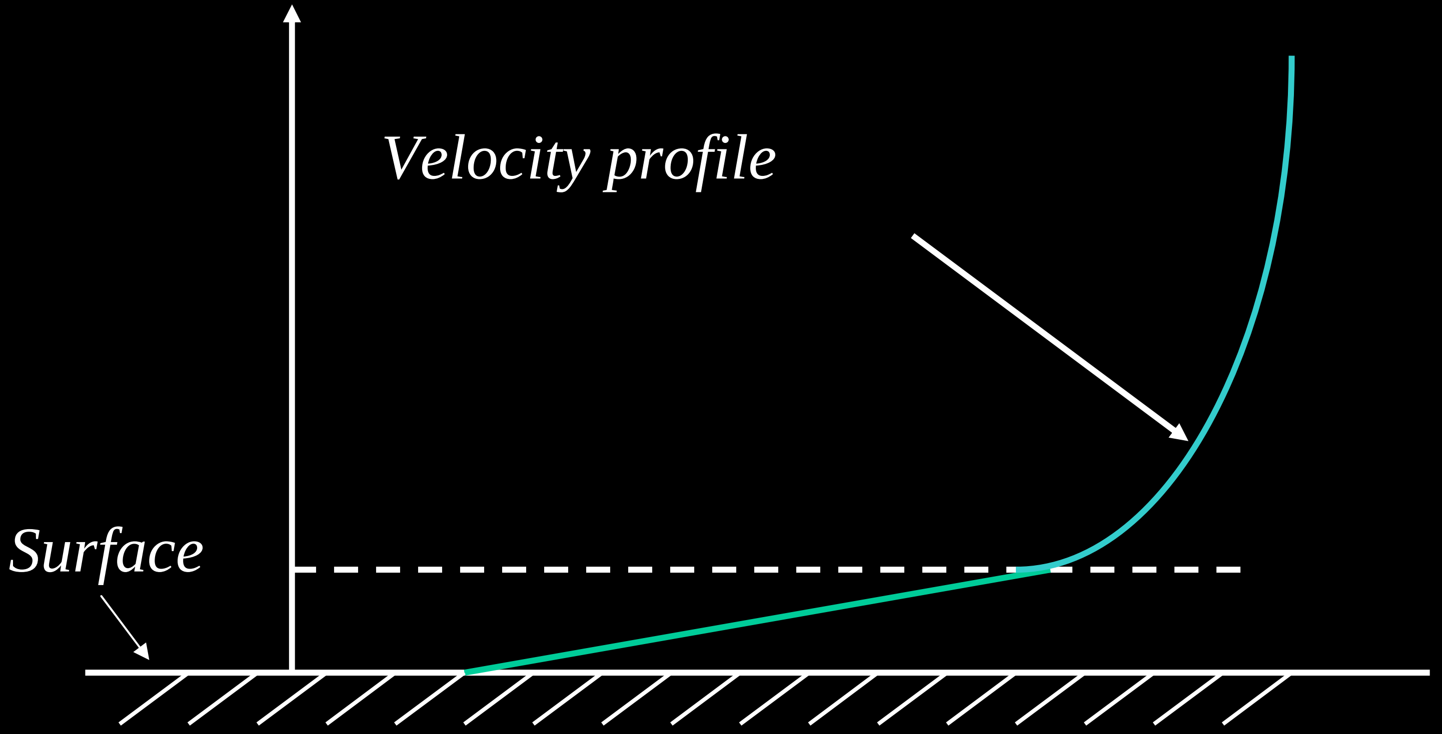
Velocity Boundary Layer

Along the solid surface, the fluid velocity varies from V_∞ to zero at the wall ⇒ velocity boundary layer δ



Near Wall Treatments

- The boundary layer is a very complex region of high velocity gradient and diffusion dominated development.
- To model it precisely would necessitate an extremely fine grid.
- An empirical relationship is therefore used to describe the shape of the boundary layer so that only one grid cell near the wall is required

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Near Wall Treatments

- Friction velocity $$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$
- Distance $$y^+ = \frac{\rho u_\tau y}{\mu}$$
- Velocity $$u^+ = \frac{u}{u_\tau}$$

Near Wall Treatments

- Relationship for smooth walls

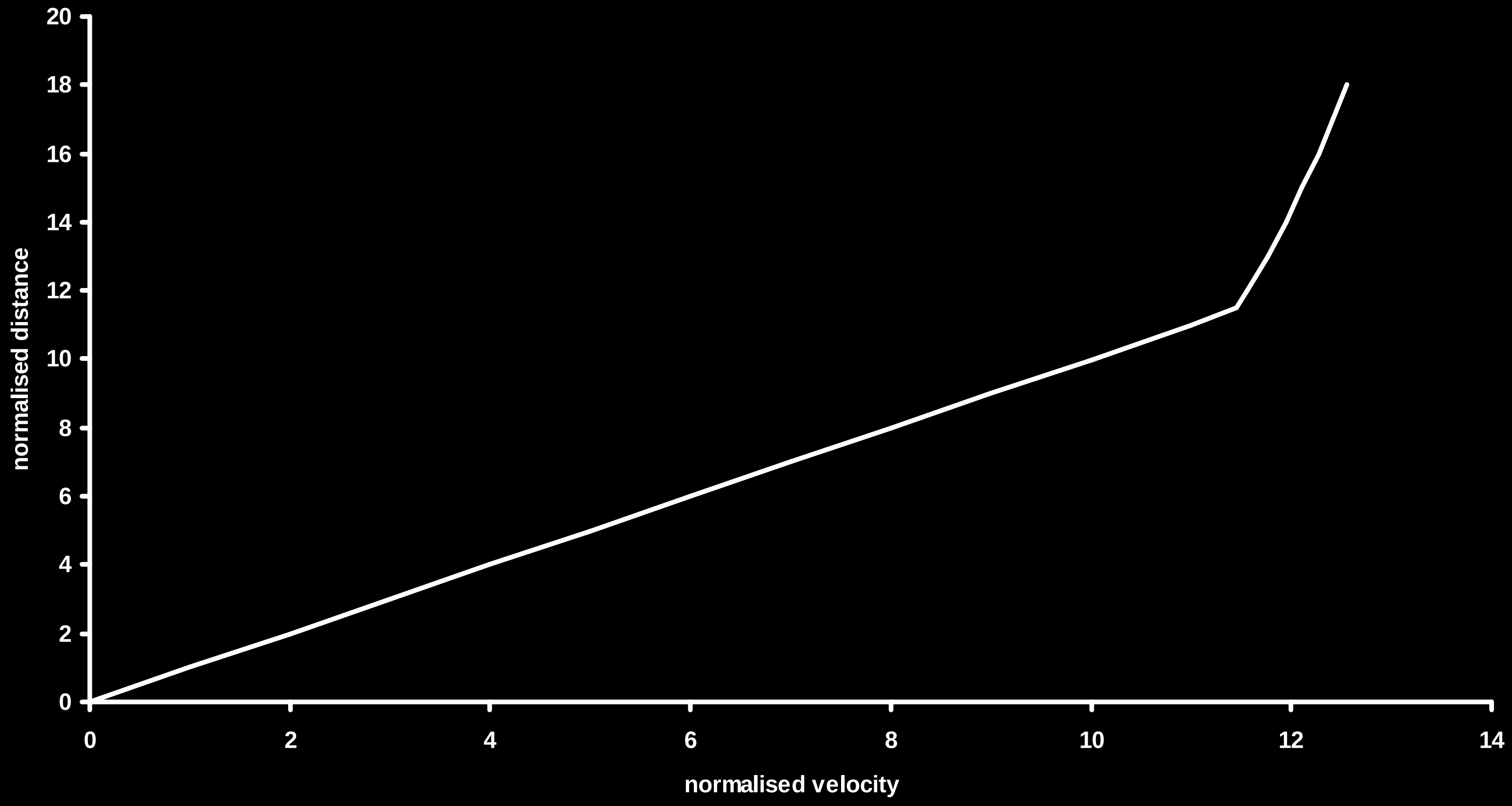
Laminar sub-layer :

$$u^+ = y^+$$

Turbulent boundary layer :

$$u^+ = \frac{1}{0.435} \ln(9y^+)$$

Graph of u^+ vs y^+



Modelling

- Fine grid to resolve thermal and velocity gradients
- Not too fine
- For KE turbulence model, first cell centre MUST be outside laminar sub-layer

First Cell

- Can find y^+ from results in first cell.
- Take velocity parallel to wall

  u
- Get half-cell height

  y
- Now, product of these:

  y u

First Cell

- Product u^+y^+ can be found

$$u^+y^+ = \frac{\rho u y}{\mu}$$

- Must now iterate to find y^+
- If $u^+y^+ > 132$, then flow is turbulent

Heat Transfer

- For smooth walls, heat transfer rate per unit area is related to the 'Stanton number':

$$St = \frac{\dot{Q}_w}{\rho U C_p \left(T - T_w \right)}$$

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Heat Transfer

- Laminar flow

$$\mathrm{St} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}$$

$$\dot{\mathrm{Q}}_{\mathrm{w}} = \frac{\mathrm{k}}{\mathrm{y}}\left(\mathrm{T} - \mathrm{T}_{\mathrm{w}}\right)$$

- Conduction into the fluid

Heat Transfer

- Turbulent flow
- Use Taylor-Prandtl analogy (heat flux is proportional to momentum flux)

$$\mathrm{St} = \frac{s}{0.9\left(1 + s^{1/2} P_j\right)}$$

Heat Transfer

- Turbulent Prandtl number = 0.9

$$s = \frac{\tau_w}{\rho u^2}$$

$$P_j = 9\left(\frac{Pr}{0.9} - 1\right)\left(\frac{0.9}{Pr}\right)^{1/4}$$

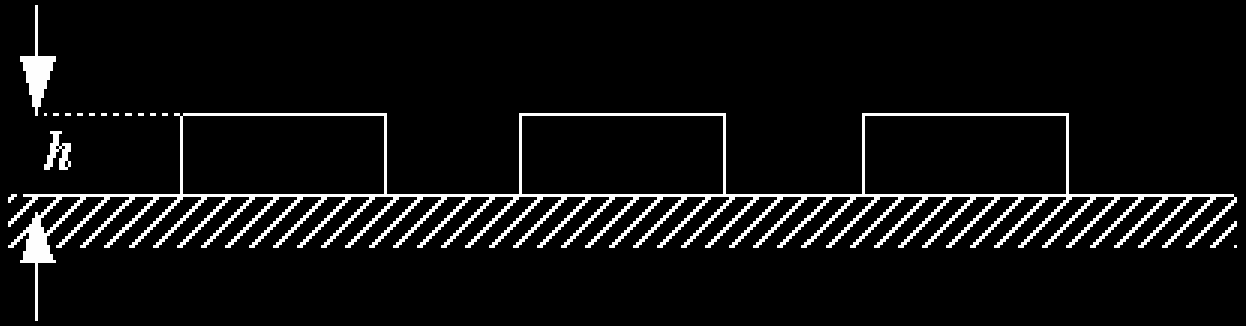
- Jayatilleke’s sub-layer resistance function

Heat Transfer

- Stanton number is determined depending on y^+ in the near wall cell
- Heat transfer rate is then determined based on temperature difference
- This heat is transferred from solid cell to fluid cell

Near Wall Treatments

- Rough walls



$$\mathrm{Re}_r = \frac{\rho h u_\tau}{\mu}$$

Friction

- Re_r =‘roughness Reynolds number’
- If $Re_r < 3.3$ treat as smooth, otherwise:

$$\frac{u}{u_\tau} = \frac{1}{0.435}\ln\left(\frac{29.7y}{h}\right)$$

- Calculate u_τ and hence τ_w (wall shear stress)